MATH1520AB 2021-22 Quiz 9 (week 13)

Full marks: 10 marks
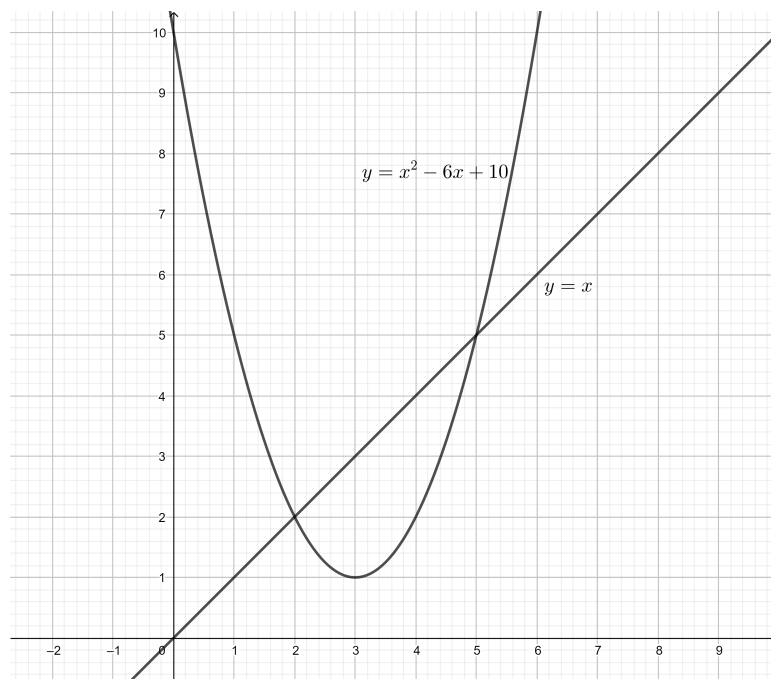
Time allowed: 15 minutes

1. Given two curves: $y = x^2 - 6x + 10$ and $y = x$

   (a) Sketch the region enclosed by the curves.

   (b) Find the points of intersection between the two curves.

   (c) Find the area of the enclosed region.

   Answer.

   (a)

   (b)

$$x^2 - 6x + 10 = x$$
$$x^2 - 7x + 10 = 0$$
$$(x-2)(x-5) = 0$$
$$x = 2 \text{ or } 5$$

   The points of intersection between the two curves are $(2,2)$ and $(5,5)$.

   (c) $Area = \displaystyle\int_2^5 [x - (x^2 - 6x + 10)]dx = \int_2^5 (-x^2 + 7x - 10)dx = \left[-\frac{x^3}{3} + \frac{7x^2}{2} - 10x \right]_2^5 = 4.5$

2. Compute

(a) $\displaystyle\int_0^{+\infty} e^{-2x}\,dx$

(b) $\displaystyle\int_1^{+\infty} \frac{3}{x^{\frac{5}{2}}}\,dx$

Answer.

(a) $\displaystyle\int_0^{+\infty} e^{-2x}\,dx = \lim_{b\to+\infty}\int_0^b e^{-2x}\,dx = \lim_{b\to+\infty}\left[\frac{e^{-2x}}{-2}\right]_0^b = \lim_{b\to+\infty}\left[\frac{e^{-2b}}{-2}\right] - \left[\frac{e^{-2(0)}}{-2}\right] = \frac{1}{2}$

(b) $\displaystyle\int_1^{+\infty} \frac{3}{x^{\frac{5}{2}}}\,dx = 3\lim_{b\to+\infty}\int_1^b \frac{1}{x^{\frac{5}{2}}}\,dx = 3\lim_{b\to+\infty}\left[-\frac{2}{3}x^{-\frac{3}{2}}\right]_1^b = 3\lim_{b\to+\infty}\left[-\frac{2}{3}b^{-\frac{3}{2}}\right] - 3\left[-\frac{2}{3}1^{-\frac{3}{2}}\right] = 2$